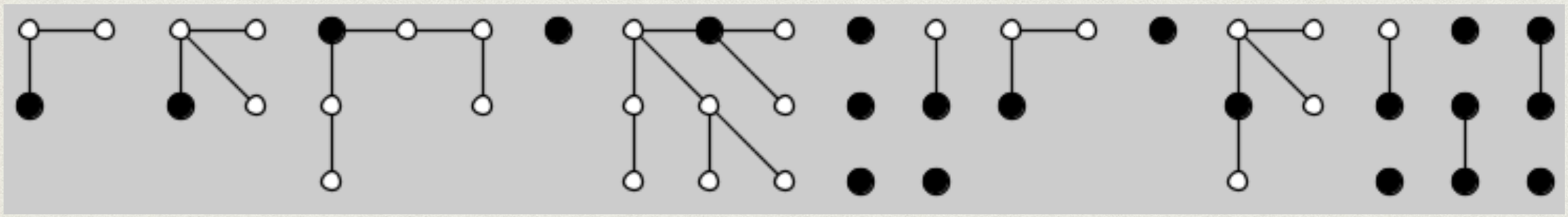
PRECISE AND CONCISE GRAPHICAL REPRESENTATION OF THE NATURAL NUMBERS

David W. Matula and Zizhen Chen
{matula, zizhenc}@smu.edu
Southern Methodist University

A GRAPHIC IS WORTH A THOUSAND DIGITS



NAMING NUMBERS

Cultural

五十

ごじゅう

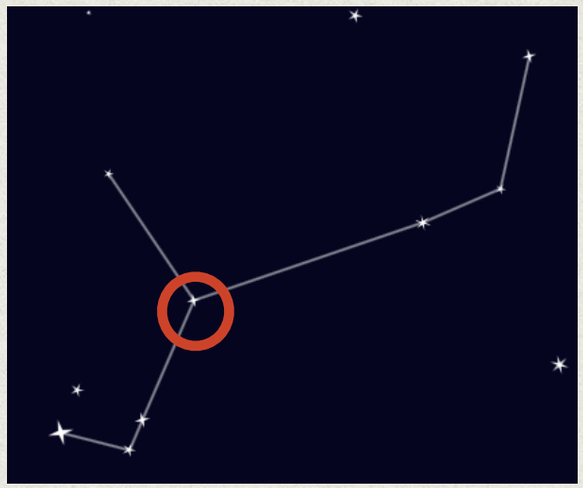
오십

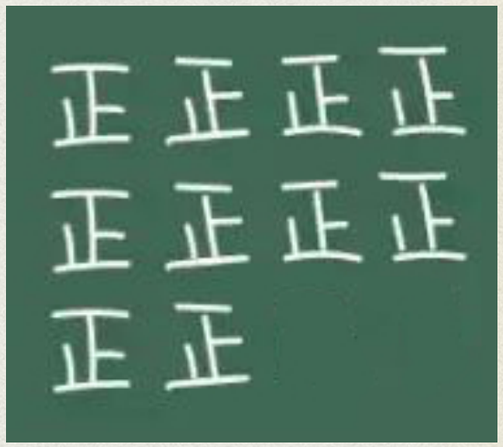
पचास

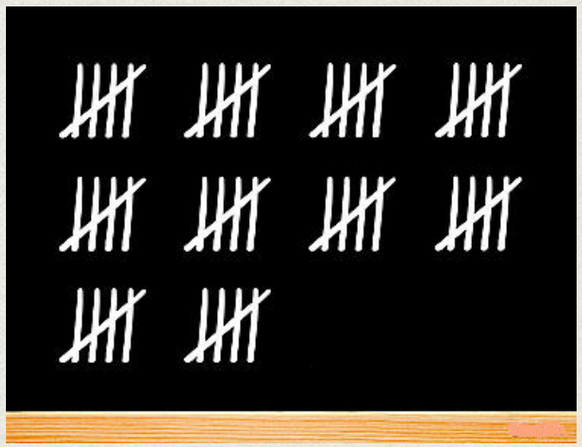
L

50

Natural







What's so special about "50"?

(It's a round number??)

五十
ごじゅう
오십
पचास

L

Why is divisible by 10 so special?

ARITH SYMPOSIUM

From 1st to 26th

NAMING NUMBERS

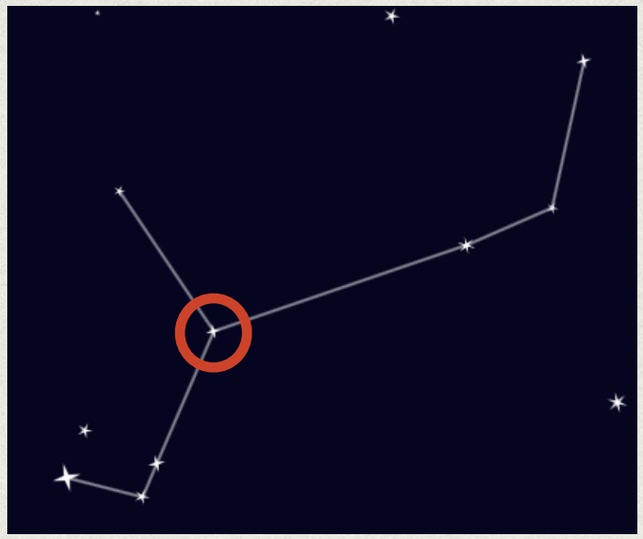
Cultural

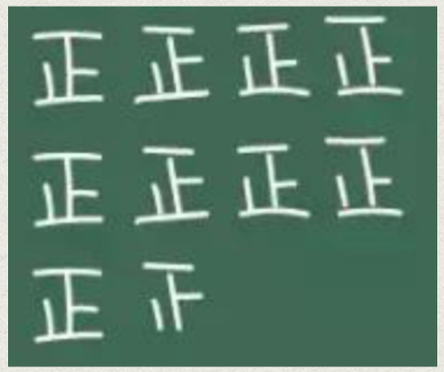
四十九	五十
よんじゅ	ごじゅう
사십구	오십
उनचास	पचास
XLIX	L
49	50

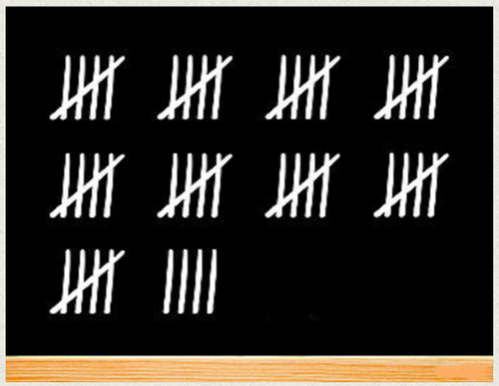
Step from 49 to 50

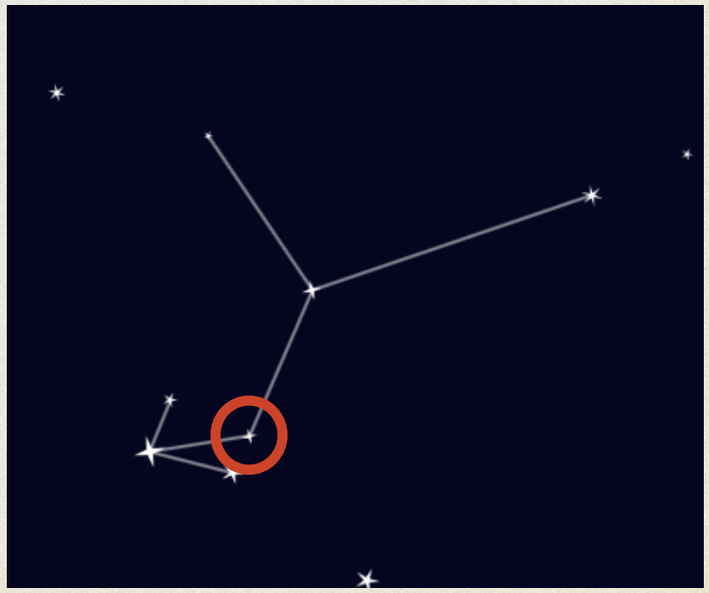
(Protocol or Obvious??)

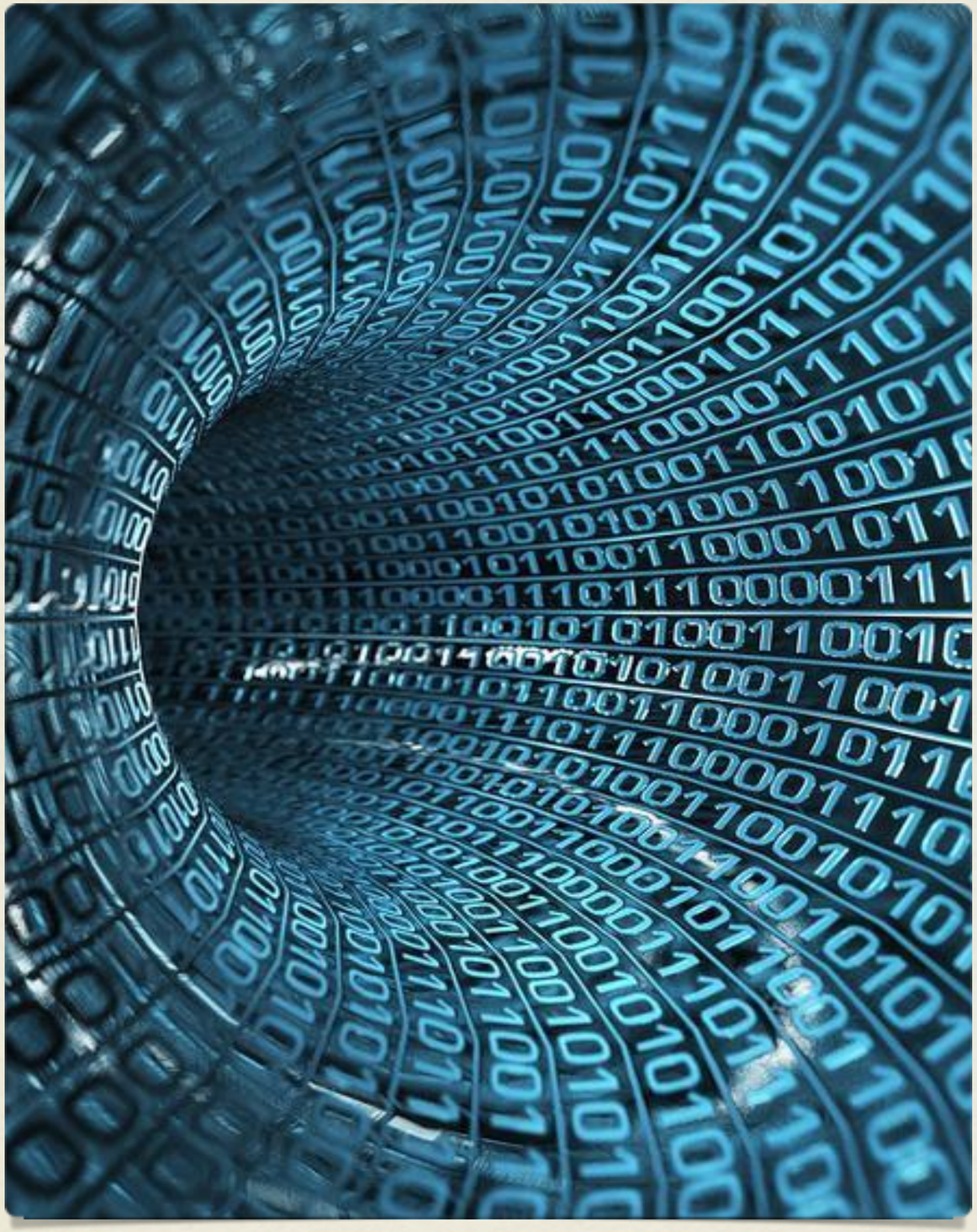
Natural











Digit (bit) strings suggest??

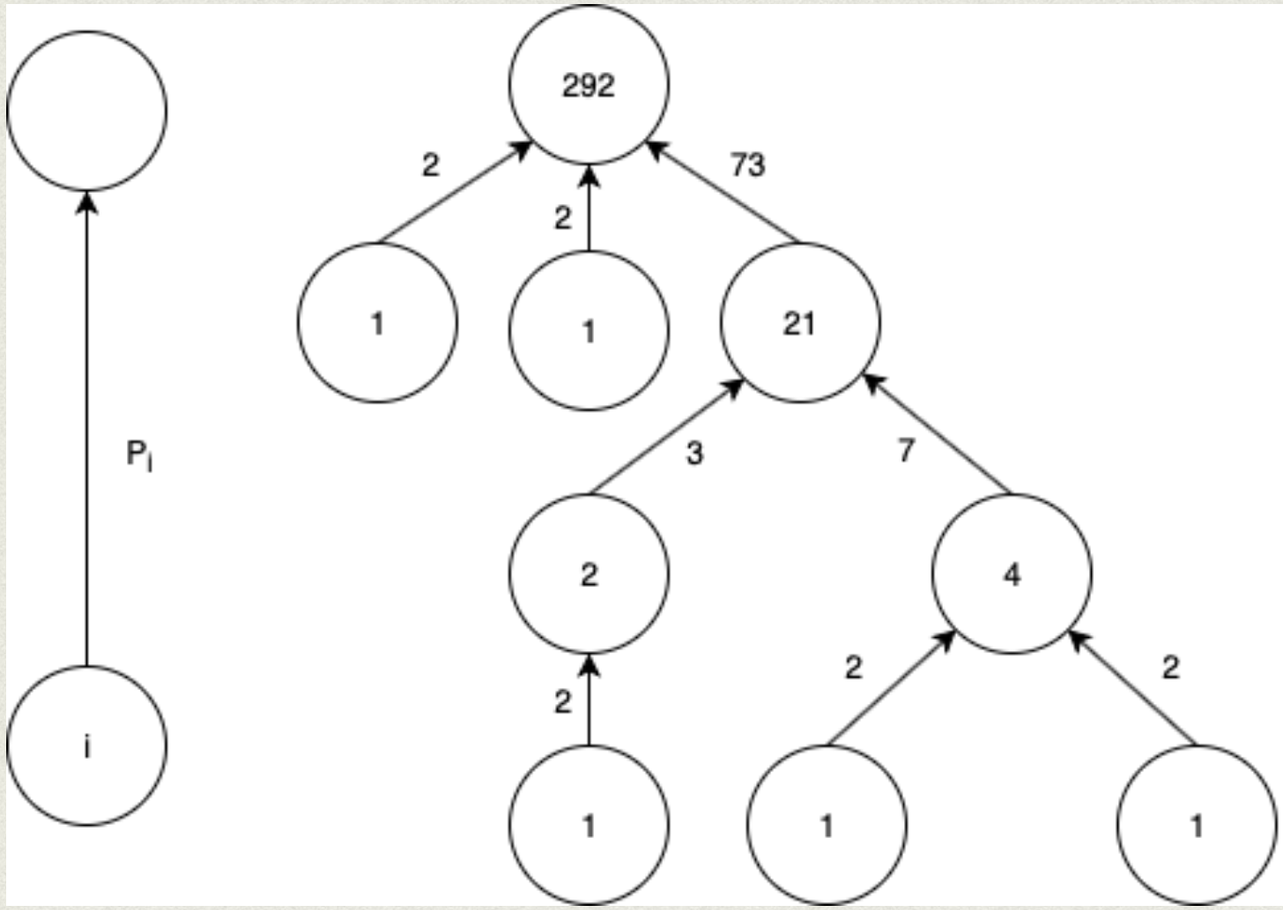
ROOTED TREES
NATURAL NUMBERS
ONE *to* ONE !

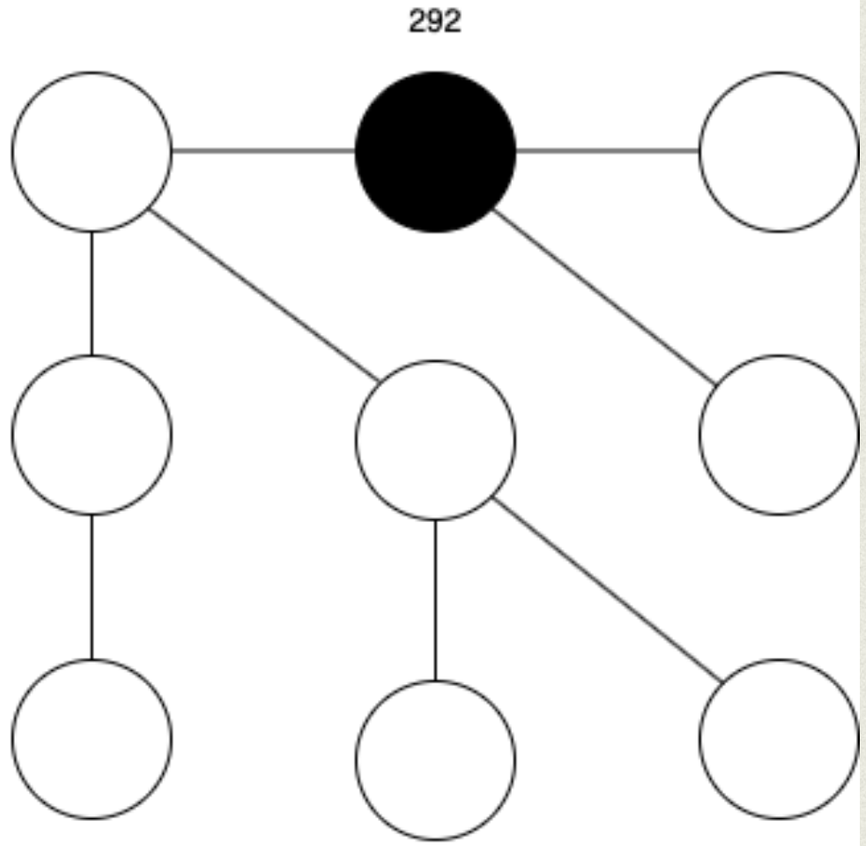
Fundamentals of Arithmetic

- Theorem: Unique Prime Factorization
- Operation: Counting (i th prime p_i)
- Procedure: Recursion (finite stopping rule)

ONE-TO-ONE CORRESPONDENCE

A Natural Procedure Over Natural Numbers





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Let's take a look...

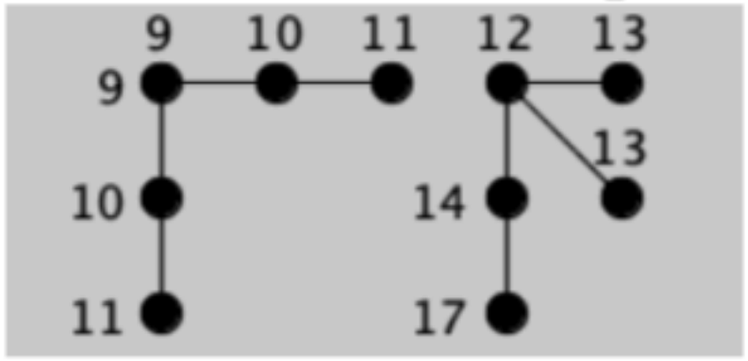
CONCISE	Structural-e.g. Digital 7 (linear)	Number Fonts
	Artistic-e.g.Chinese, etc. (2D)	
PRECISE	Integer <=> One Tree	
	Rational Fraction <=> Two Trees	
	Continued Fraction <=> Sequence of Trees	
	Reals by "Best Rational Approximation"	

STRUCTURAL FONTS

Decimal Digits vs. Rooted Trees

(a) digits in the Digital-7 font

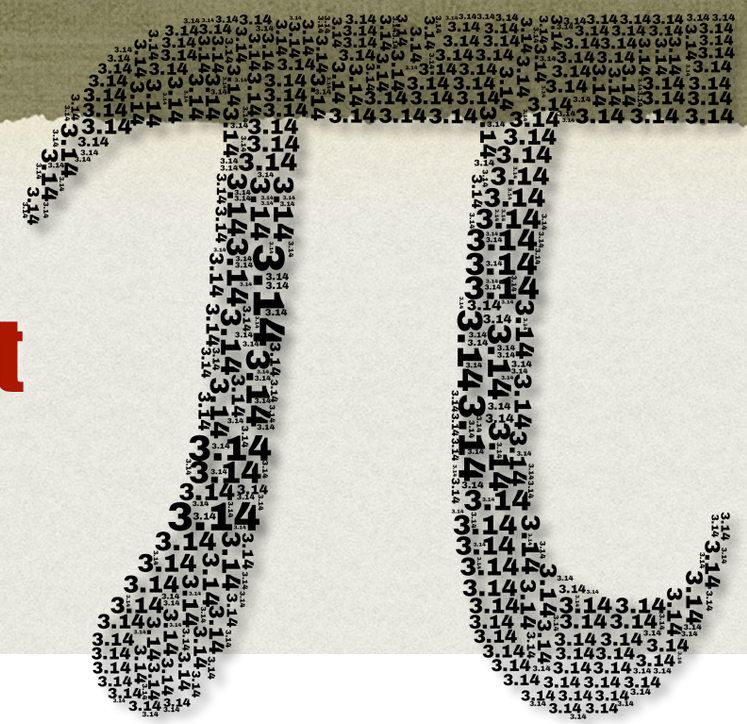
(b) counts $1, 2, 3, \ldots, 9$ in a square grid font



(c) compressed square grid font for selecting counts 9-14 and 17

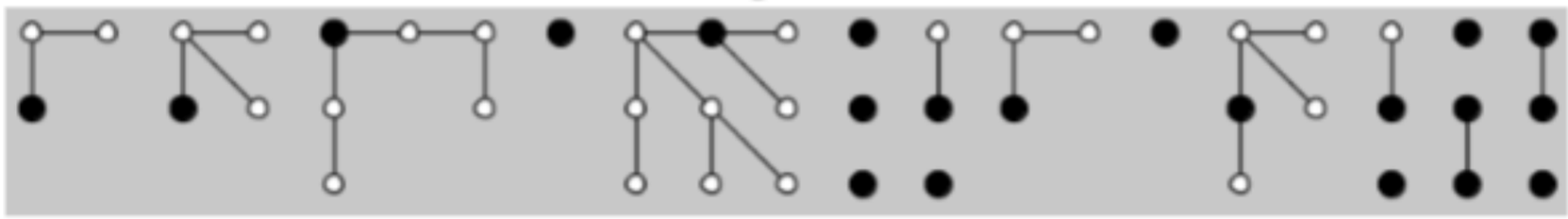
FIRST 21 PARTIAL QUOTIENTS

Everyone looks at



3.7.15.1.292.1.1.1.2.1.3.1.14.2.1.1.2.2.2.2.1

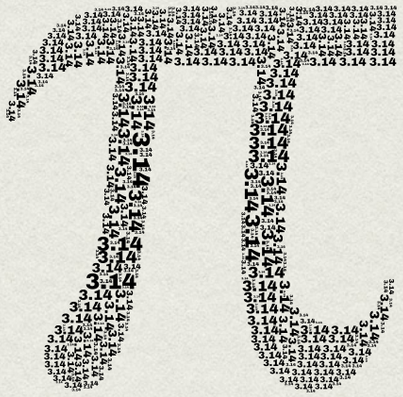
(a) Digital-7 font



(b) square grid font

RATIONAL FRACTION FORM

Continued Fraction (10 partial quotients)

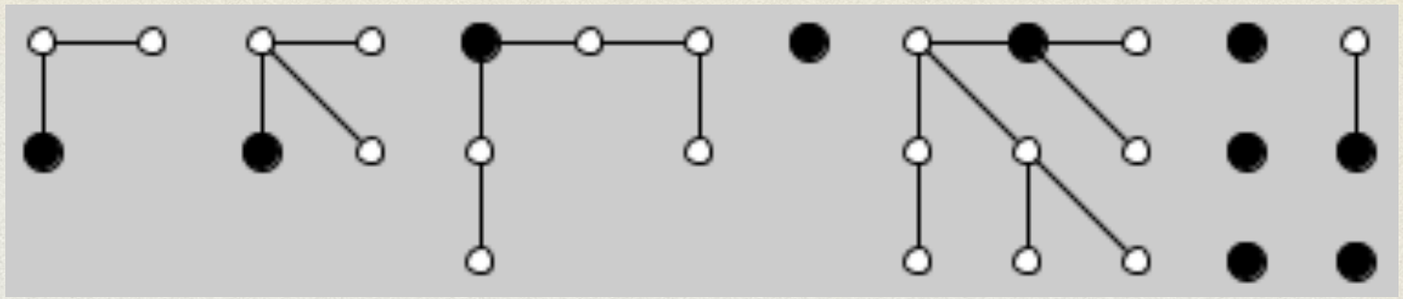


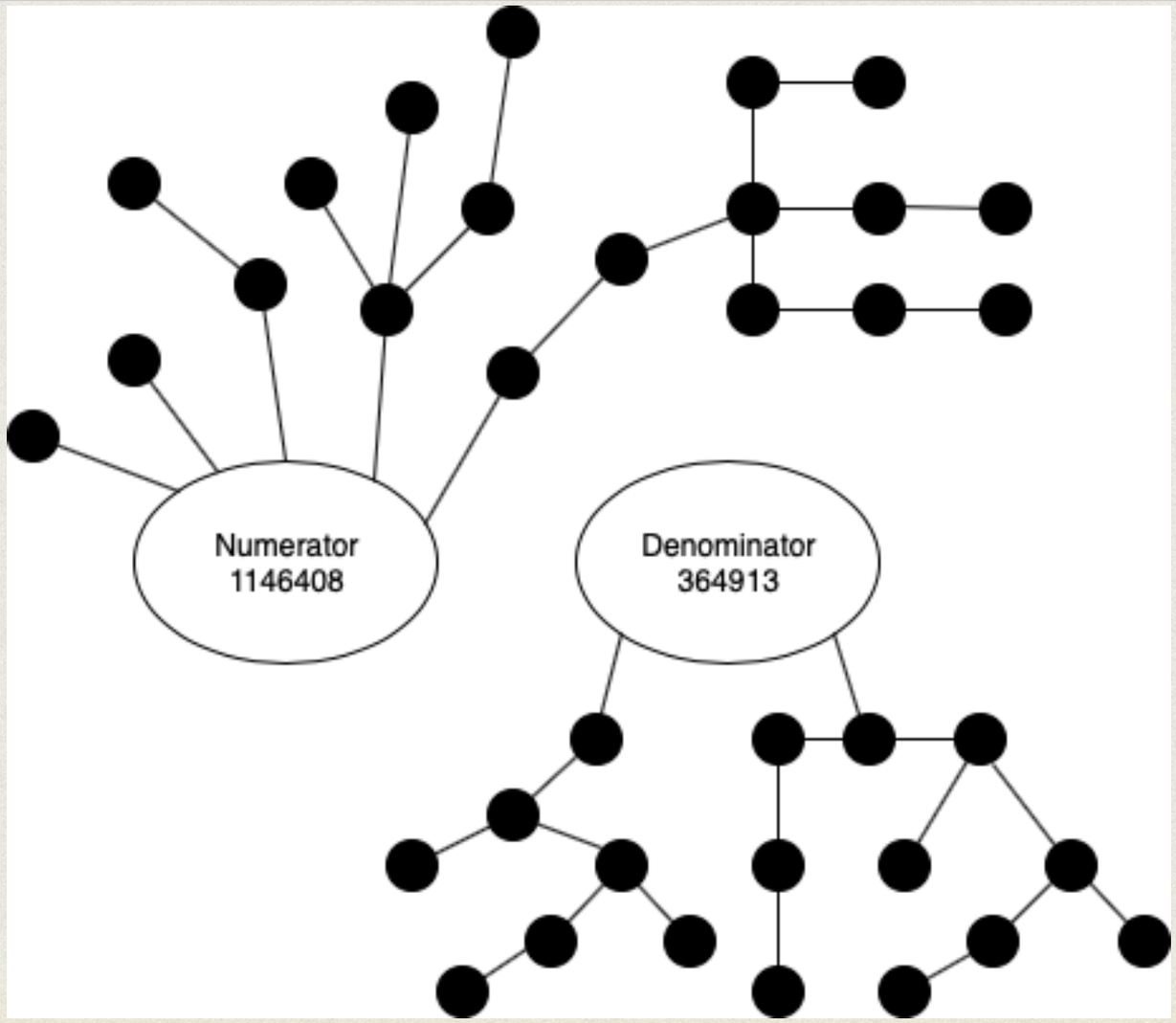
Rational Fraction (reduced)

1146408/364913

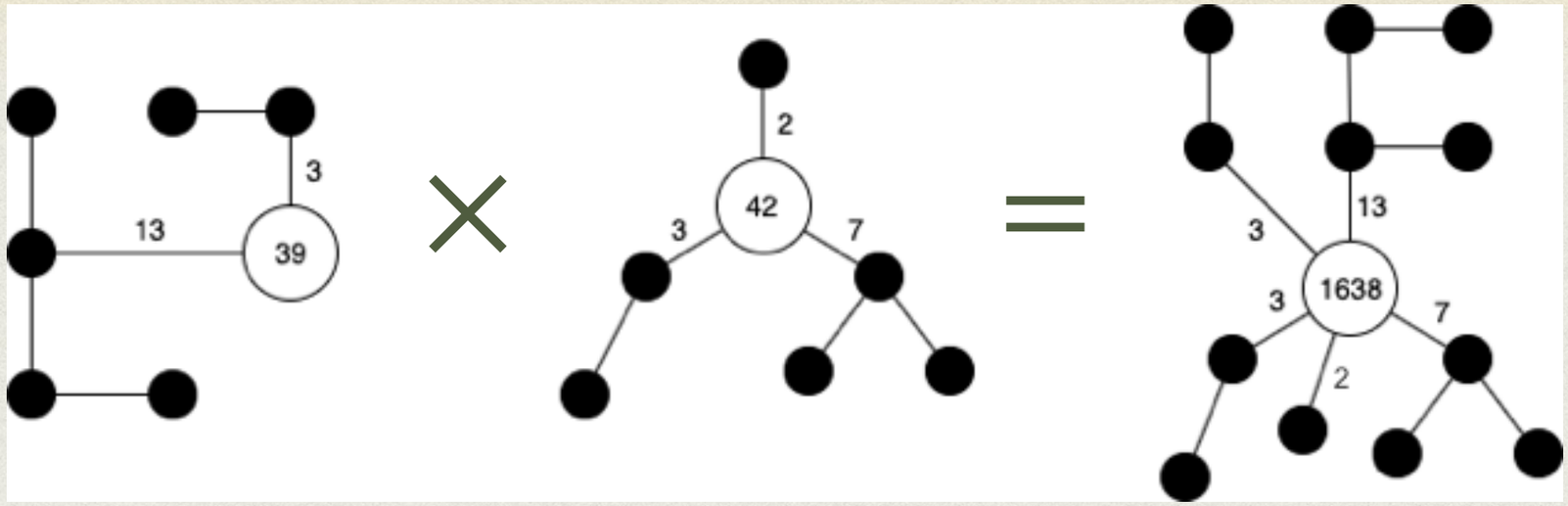
=3.14159265358…

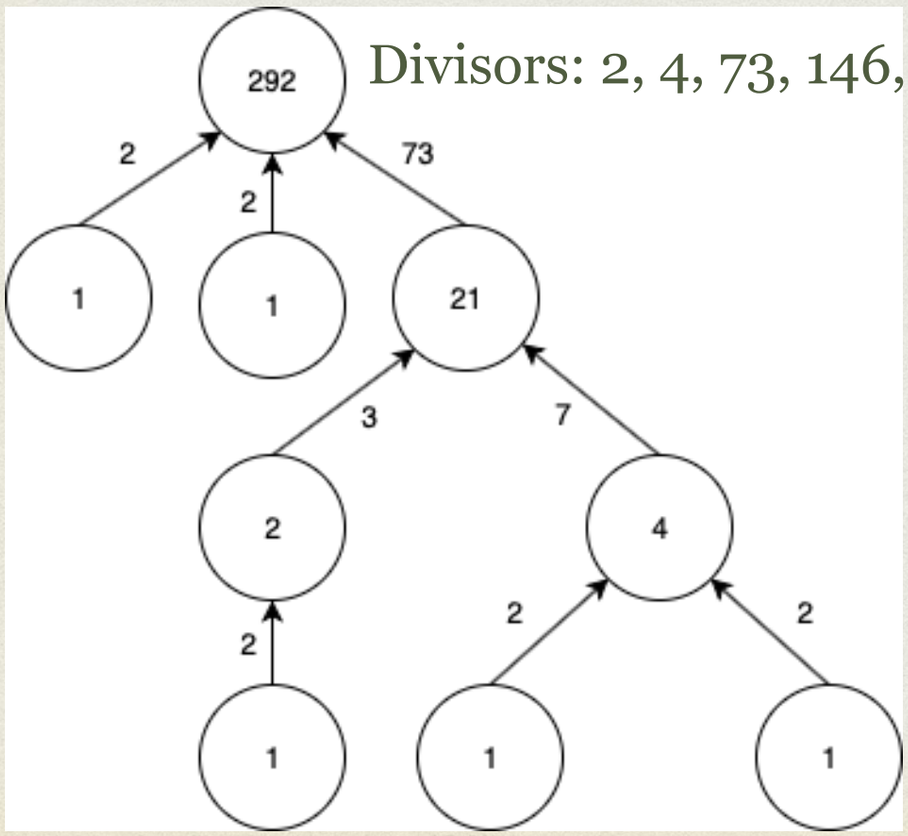
“correct digits”



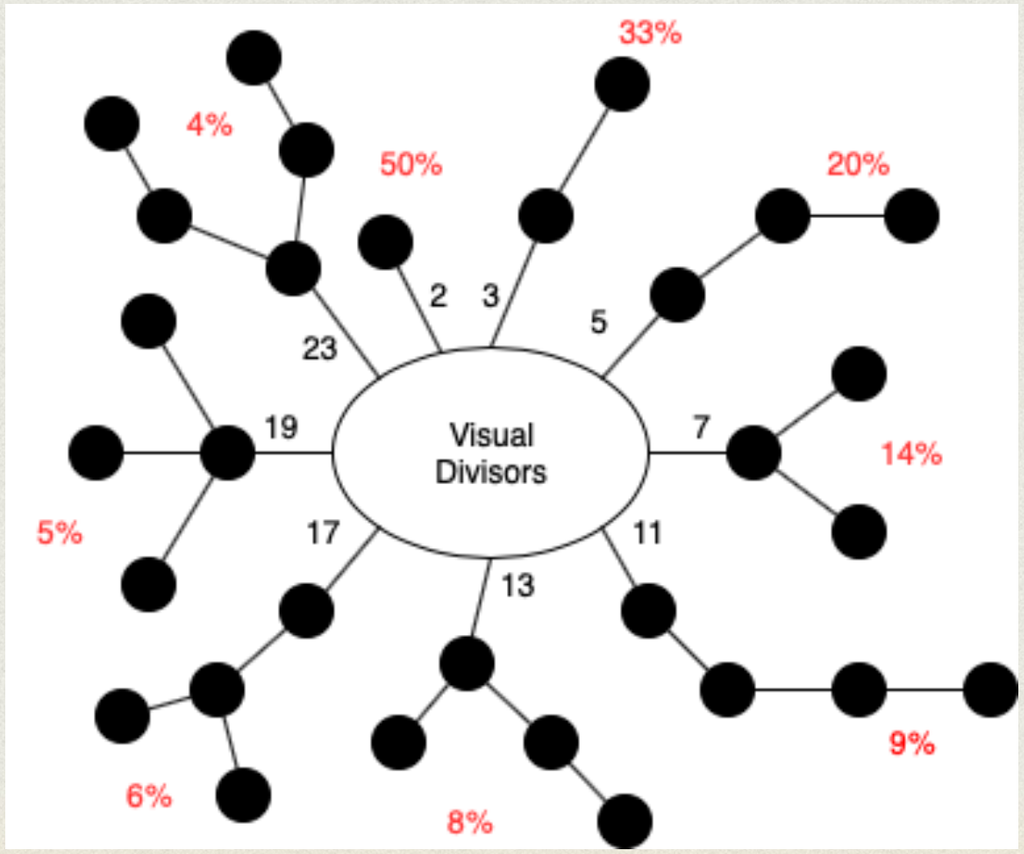


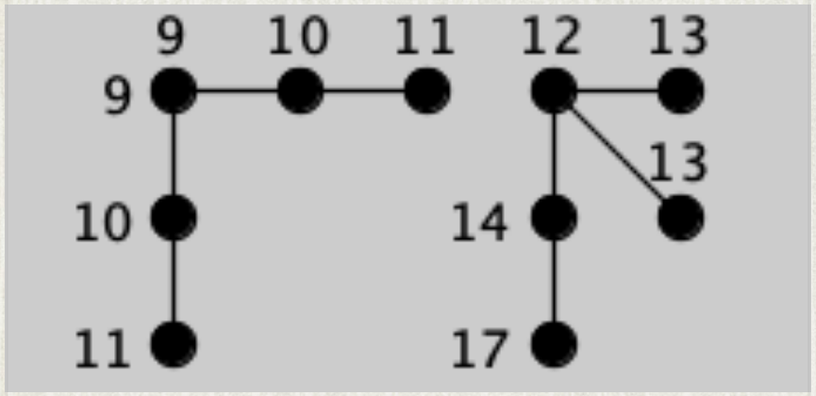
MULTIPLICATION IS VISUAL

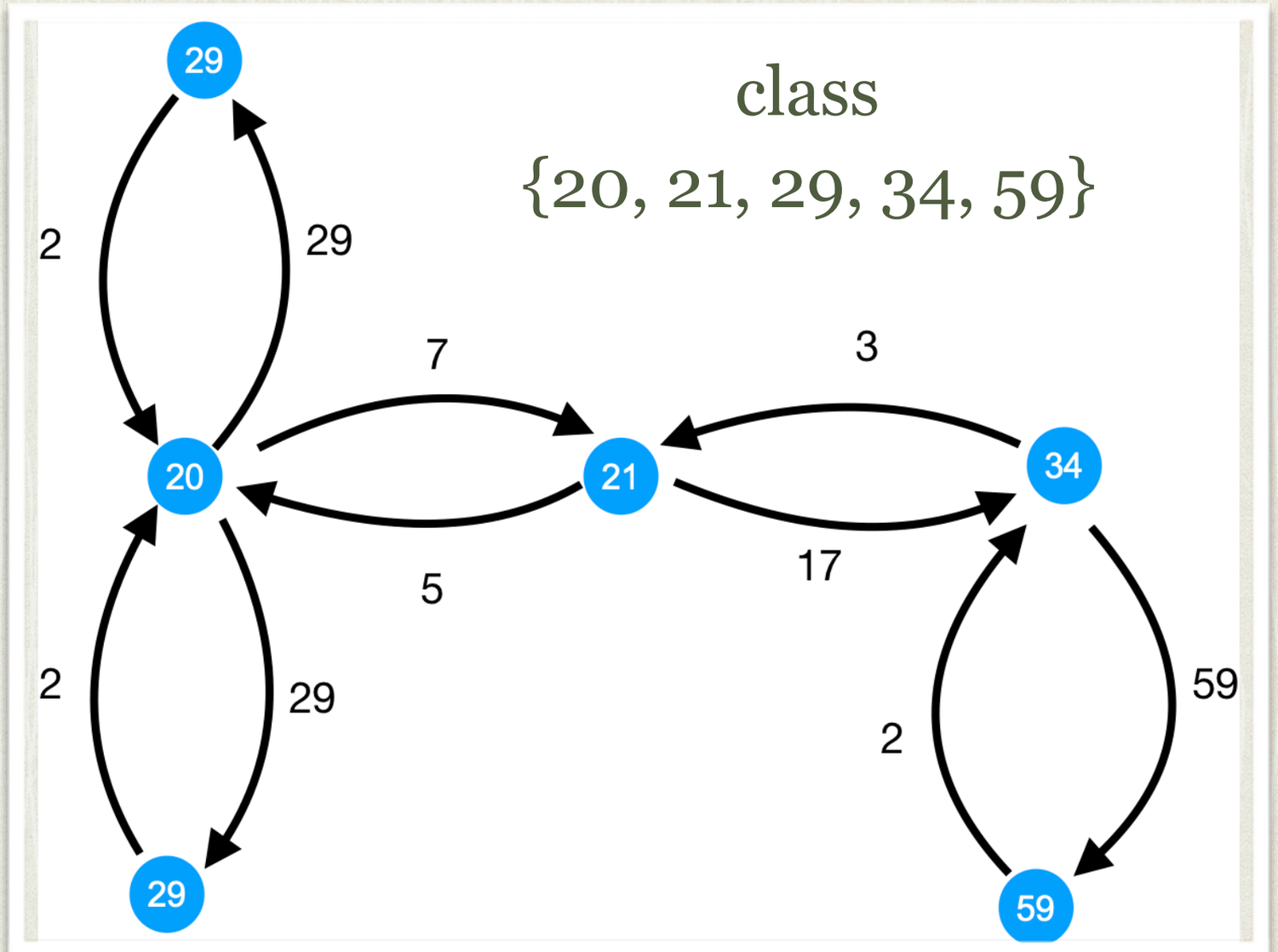




Divisors: 2, 4, 73, 146, 292

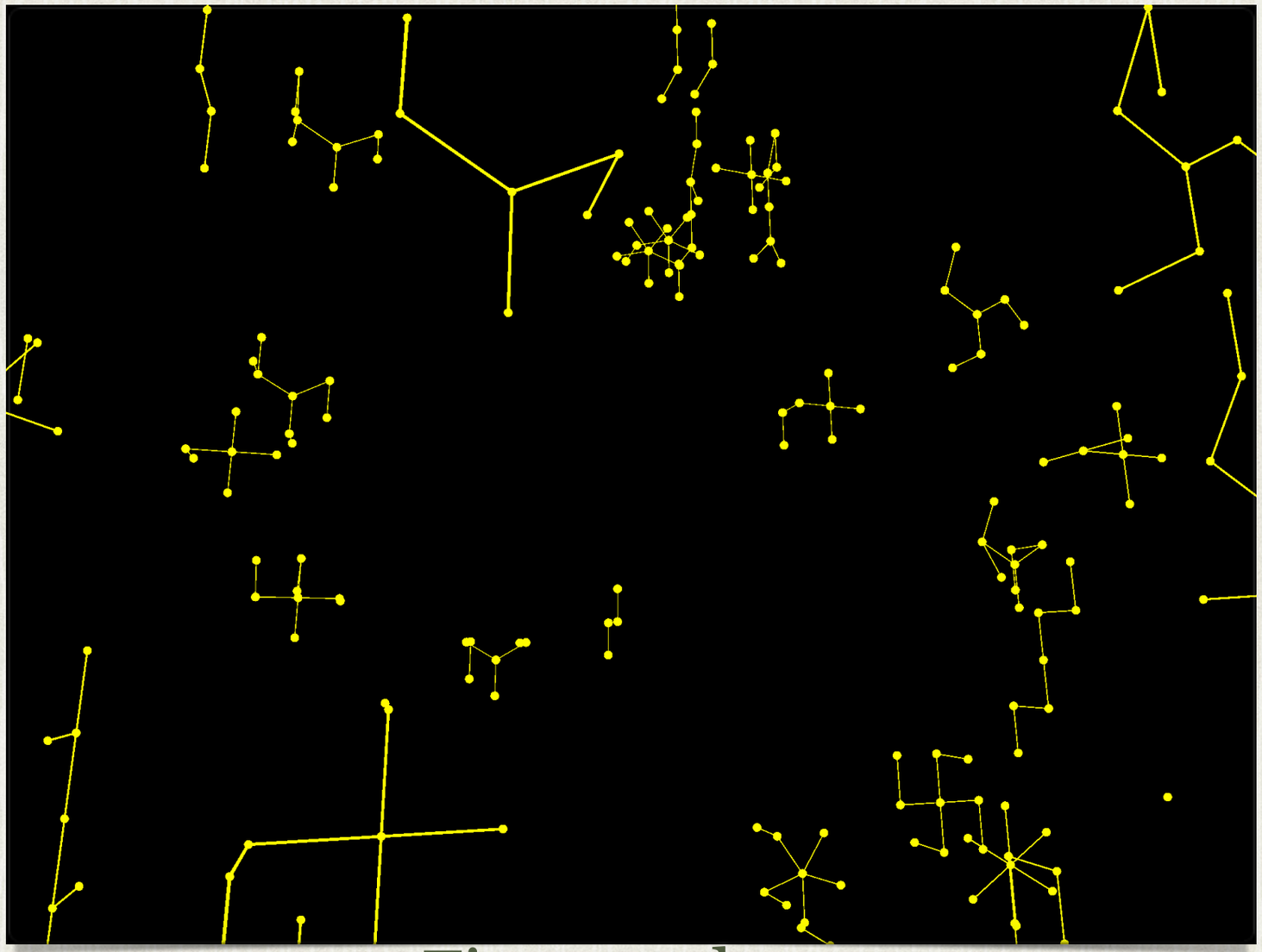






EQUIVALENCE RELATION

$[j(p_j)]$ **R** $[i(p_j)]$



First 40 classes

$$[j(p_i)] \mathbf{R} [i(p_j)]$$